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SOLVING THE TRAVELLING SALESMAN PROBLEM WITH A HOPFIELD-TYPE NEURAL NETWORK

1. Introduction

Hopfield-type neural networks [5] composed of highly-interconnected analog elements (neurons) can be successfully used in solving optimization problems. Structure of a network and weights of connections between neurons depend on the specific constraints of a problem. For each neuron in the network the so-called input and output potentials can be defined, denoted by u and v, respectively. In the Hopfield model, the function of response is usually S-shaped. In this paper

$$v(u) = 1/2[1 + \tanh(\alpha u)] \tag{1}$$

where α is a gain of that function. For sufficiently big values of α, v is of binary character, i.e. approximately

$$v(u) = \begin{cases} 0 & \text{if } u < 0 \\ 1 & \text{if } u > 0. \end{cases}$$

In a network composed of m neurons a function of energy E of the network is in general of the following form [5, 6]:

$$E = -\frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} t_{ij} v_i v_j \tag{2}$$

where t_{ij} $(i, j = 1, \ldots, m)$ is weight of a connection between the output of the j-th neuron and the input of the i-th one. All t_{ij} form a matrix of connection weights. They can be positive (excitatory stimulus) or negative (inhibitory stimulus) or equal to zero, i.e. there is no connection from neuron j to neuron i. The input potential u_i of the i-th neuron is defined by the

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equation

$$u_i = -\frac{\partial E}{\partial v_i} \quad (i = 1, \ldots, m). \tag{3}$$

From (2) and (3) we obtain

$$u_i = \sum_{j=1}^{m} t_{ij} v_j \quad (i = 1, \ldots, m). \tag{4}$$

The above rules were exploit by various authors in attempts to solve hard optimization problems. The greatest attention among them was probably paid to the TSP. The problem can informally be stated as follows:

DEFINITION 1. Given a set of n cities and distances between every two of them, find the closed tour for a salesman through all cities that visits each city only once and is of minimum length.

Based on the Graph Theory terminology the TSP is defined as below:

DEFINITION 2. Given a graph K_n and a symmetric matrix representing weights of edges in K_n, find the Hamiltonian cycle in K_n of minimum length (cost).

Note 1. Problem definition presented here is not the only possible version of the TSP. In other definitions, the matrix is not symmetric or not every two different cities are connected by an edge. Some alternative definitions, as well as main, well known theoretical results concerning the computational complexity of various versions of the TSP can be found in [13].

Note 2. Since, the two above mentioned definitions are equivalent, and due to some "tradition" associated with formulating the TSP is terms of "route across cities", we would rather use terminology from Definition 1.

Due to the presumable non-polynomial complexity of the TSP, conventional approaches to solving the problem are based either on the extensive search methods or on heuristics. The most popular and one of the best heuristical methods are those presented in [8], [3], [4] or [12].

Our solution to the problem is based on the papers [15], [9] and [11]. Actually, the idea of network evolution as well as the way of choosing starting point of the simulation are identical to the ones used for the N-Queens Problem (NQP) in the last two of above cited papers.

The fact that these two problems are well suited to the method proposed is encouraging and stimulating for future research and development of the method.

In this paper, any syntactically valid solution of the TSP (Hamiltonian cycle in a city graph) will be treated as *solution* of the problem, and solutions

that minimize length of the tour will be called *best solutions*. Obviously, for any solution there exist $2n - 1$ other solutions of the same length which differ from one another in the starting city or direction of the tour. For the sake of simplicity, all of them would be treated as the same solution.

Solving optimization problems with the Hopfield network requires careful and adequate choice of the energy function, i.e. weights t_{ij}. Function E must be determined in such a way that its minima correspond to solutions of the problem considered.

In this paper E is of the form

$$E = E_1 + E_2 \tag{5}$$

where

$$E_1 = \frac{A}{2}\sum_{x=1}^{n}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} v_{xi}v_{xj} + \frac{B}{2}\sum_{i=1}^{n}\sum_{x=1}^{n}\sum_{\substack{y=1\\ y\neq x}}^{n} v_{xi}v_{yi} + \frac{C}{2}\Big(\sum_{x=1}^{n}\sum_{i=1}^{n} v_{xi} - (n+\sigma)\Big)^2 \tag{5.1}$$

and

$$E_2 = \frac{D}{2}\sum_{x=1}^{n}\sum_{\substack{y=1\\ y\neq x}}^{n}\sum_{i=1}^{n} d_{xy}v_{xi}(v_{y,i+1} + v_{y,i-1}). \tag{5.2}$$

In eqs. (5.1) and (5.2), n denotes the number of cities, and d_{xy} the distance between cities x and y.

The way of solving the TSP presented here, except for different choice of network parameters, differs from the classical approach in the way of updating neurons output potentials. Hopfield [5] and others ([14], [7], [1]) have used the following updating rule

$$\frac{du_{xi}}{dt} = -\frac{u_{xi}}{\tau} - A\sum_{\substack{j=1\\ j\neq i}}^{n} v_{xj} - B\sum_{\substack{y=1\\ y\neq x}}^{n} v_{yi} - C\Big(\sum_{x=1}^{n}\sum_{j=1}^{n} v_{xj} - (n+\sigma)\Big) - D\sum_{y=1}^{n} d_{xy}(v_{y,i+1} + v_{y,i-1}) \tag{6}$$

with $\tau = 1$.

In the practical computer realization, after applying the Euler method, eq. (6) was of the form

$$(7)\quad u_{xi}(t+\Delta t) = u_{xi}(t) + \Delta t\Big(-u_{xi}(t) - A\sum_{\substack{j=1\\ j\neq i}}^{n} v_{xj}(t) - B\sum_{\substack{y=1\\ y\neq x}}^{n} v_{yi}(t) -$$

$$-C\Big(\sum_{x=1}^{n}\sum_{j=1}^{n} v_{xj}(t) - (n+\sigma)\Big) - D\sum_{y=1}^{n} d_{xy}(v_{y,i+1}(t) + v_{y,i-1}(t))\Big)$$

with Δt equal to 10^{-5}.

In eq. (7), the state of neuron xi $(x, i = 1, \ldots, n)$ at time $t+1$ depends on its state at time t. In our simulations an input potential at time $t+1$ does not directly depend on its state in the previous moment. Actually, from (3) and (5)

$$(8)\quad u_{xi}(t+1) = -A\sum_{\substack{j=1\\ j\neq i}}^{n} v_{xj}(t) - B\sum_{\substack{y=1\\ y\neq x}}^{n} v_{yi}(t) -$$

$$-C\Big(\sum_{x=1}^{n}\sum_{j=1}^{n} v_{xj}(t) - (n+\sigma)\Big) - D\sum_{y=1}^{n} d_{xy}(v_{y,i+1}(t) + v_{y,i-1}(t)).$$

The above presented updating rule was also used in [15]. Yao et al. [15] reported used about 50% convergence rate to valid tours. Better convergence rate obtained in our simulations is due to a better choice of network constants.

The biggest advantage of the proposed network is its independence on the initial state (the output potentials of neurons at the beginning of a simulation test). Results of computer simulations as well as discussion on the influence of network constants on the quality of results are presented in the following sections.

2. Network description and simulation results

In computer simulations the network was represented by a matrix $V_{n\times n}$. At the end of a simulation test which converged to a solution each element v_{xi} $(x, i = 1, \ldots, n)$, representing output potential of neuron xi, was equal to either zero or 1. Moreover, elements of V fulfiled the constraints that in each row and in each column of V there existed exactly one element equal to 1. In the resulting matrix, $v_{xi} = 1$ was interpreted as that city x was in the i-th position in a salesman's tour.

The above requirements for resulting matrix V were implied by the condition that minima of (5) should correspond to solutions of the problem.

Actually, in (5.1) term multiplied by A fulfils the constraint that in each row x there exist at most one element equal to 1 (city x is visited not more than once). Similarily, term multiplied by B fulfils the same condition for

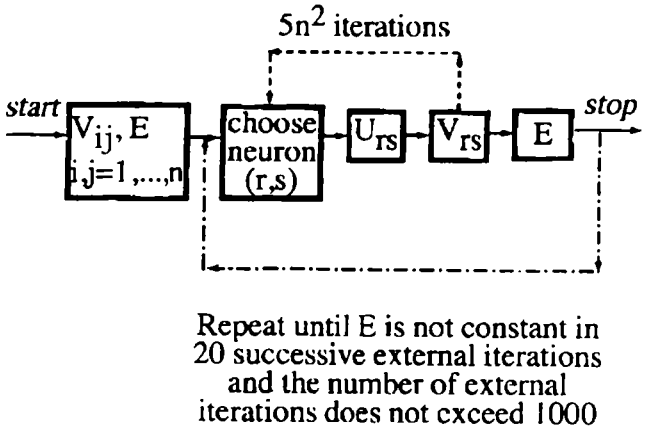


Fig. 1. A diagram of network evolution in one test

columns (at the i-th step of the tour at most one city is visited). Finally, the third term in (5.1) forces the sum of all elements of V to a value close to n, which means that the tour is composed of n steps.

Minimization of a tour length is covered by (5.2).

In a simulation test, the network starting from some energy level, slowly decreases its energy and, in the end, settles in the minimum of E. The "deeper" the minimum, the better the obtained solution (global minima of E correspond to best solutions).

A single simulation test was performed as follows (see Fig. 1):

(i) all initial output potentials v_{xi} $(x, i = 1, \ldots, n)$ were set and from (5) the starting value of energy E was evaluated,

(ii) neuron (x, i) was chosen at random and from (8) u_{xi} was calculated, and then from (1) v_{xi} was obtained.

Operation (ii) was repeated $5n^2$ times, and then a new value of E from (5) was calculated.

Every n^2 repetitions of (ii) was called *an internal iteration*. Five internal iterations composed one *external iteration*.

A simulation process terminated if the energy remained constant in *a priori* established amount of successive external iterations or, the number of external iterations exceeded the constraint for a global number of iterations and the network still did not achieve a stable state, i.e. a constant value of E.

In simulation tests four strategies for setting initial output potentials were used. In those strategies the output of each of n^2 neurons was initially set to (cf. [9], [11]):

a — random value from $[0, \beta]$,
b — random value from $[0, 1]$,
c — random value from $[1 - \beta, 1]$,
d — random value from $[0, \beta] + 1/n$,

where $\beta \approx 0.03$.

Two strategies, denoted F (Full) and P (Part) were employed for choosing neuron xi to be modified in (ii).

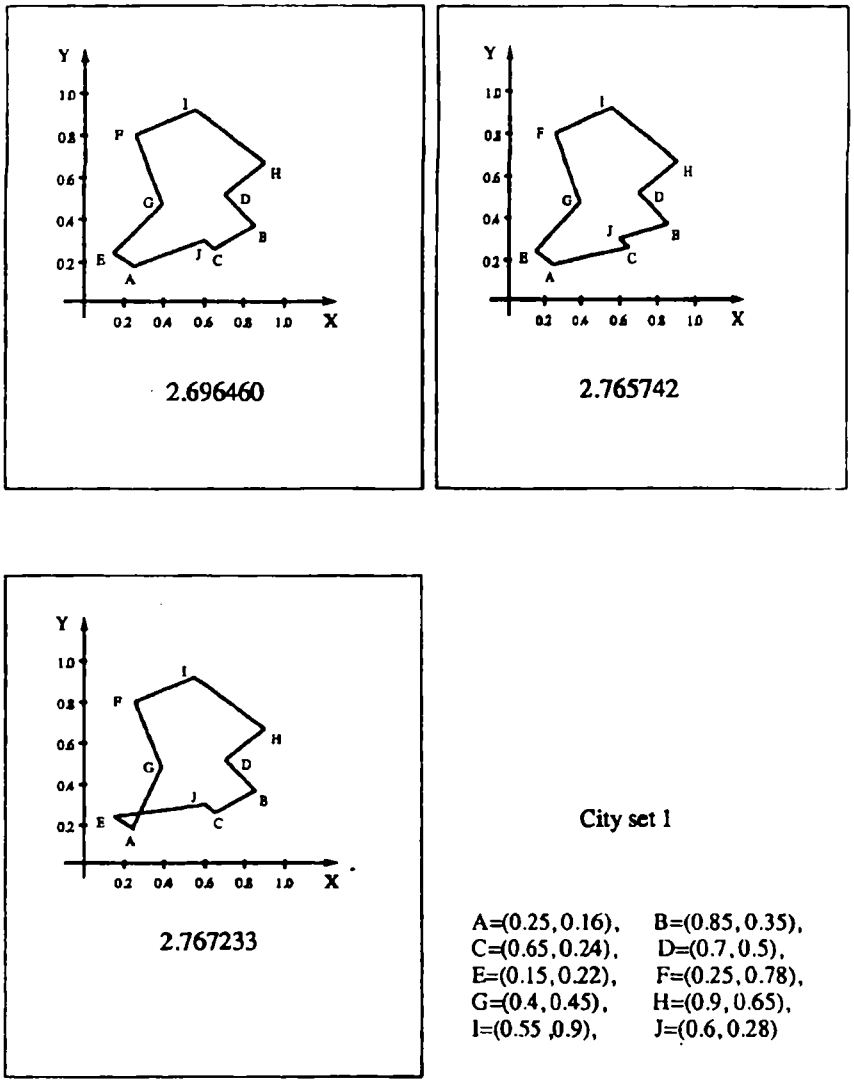
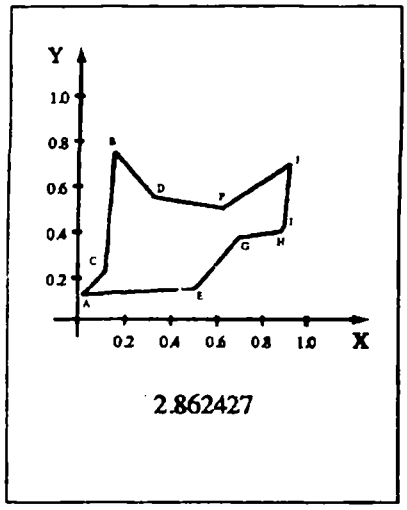
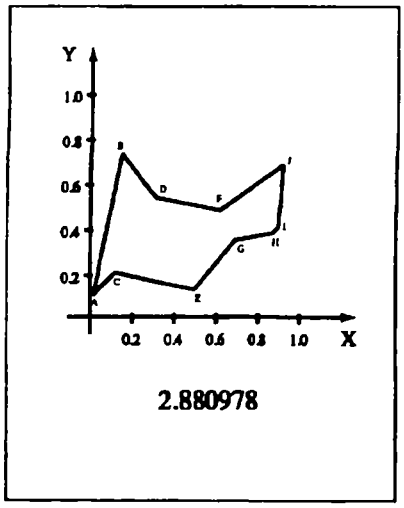


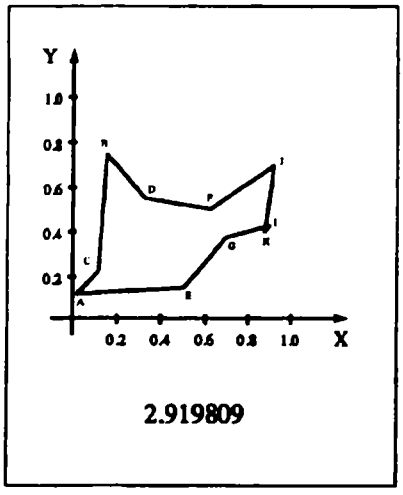
Fig. 2. City set 1: cities coordinates and three shortest tours

In case F, the choice of neurons to be modified in the internal iteration was completely random. In case P, in every internal iteration a permutation of numbers $1, \ldots, n^2$ was randomly chosen, and neurons were modified according to that permutation.

Simulations were based on three 10-element data sets. The first one was taken from [5], next one from [1] and the last one was similar to one of the sets used in [2]. Three "shortest" tours for each data set and coordinates of cities are presented in Figs. 2–4.







City set 2

A=(0.025, 0.125),	B=(0.15, 0.75),
C=(0.125, 0.225),	D=(0.325, 0.55),
E=(0.5, 0.15),	F=(0.625, 0.5),
G=(0.7, 0.375),	H=(0.875, 0.4),
I=(0.9, 0.425),	J=(0.925, 0.7)

Fig. 3. City set 2: cities coordinates and three shortest tours

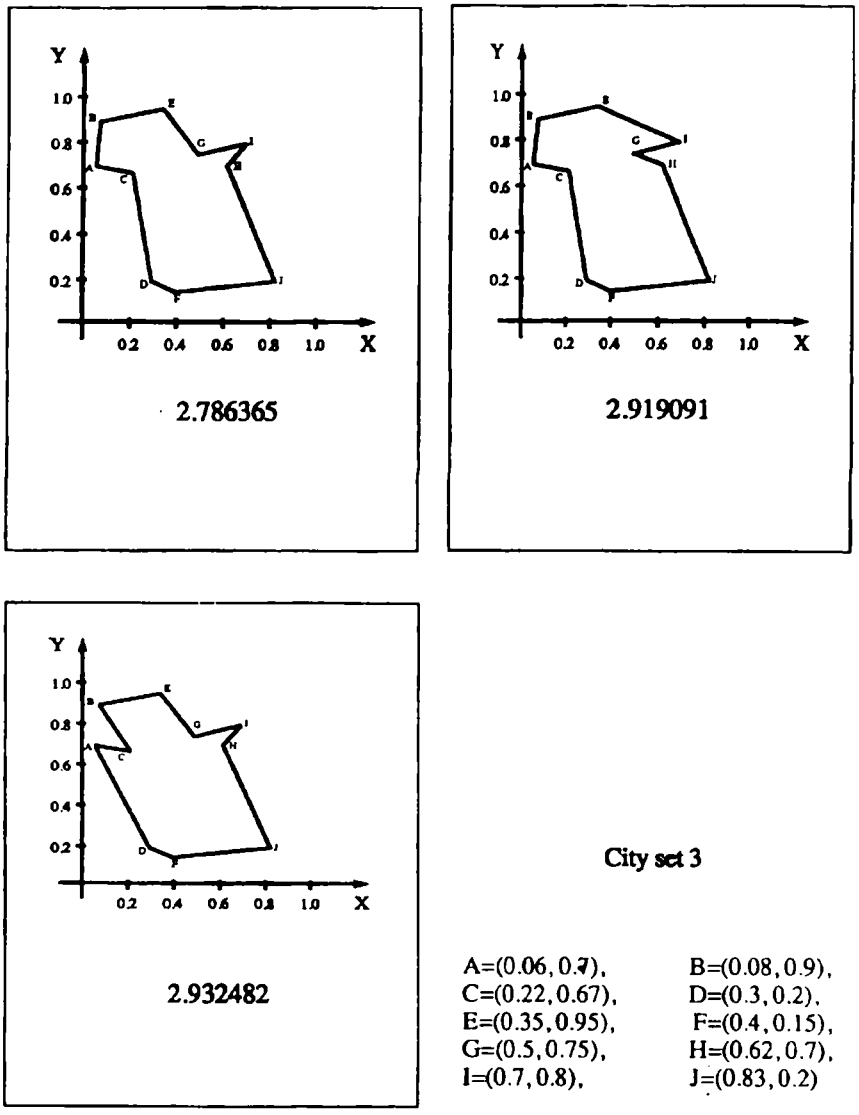


Fig. 4. City set 3: cities coordinates and three shortest tours

As previously stated, the crucial point of network design is the proper choice of connection weights or, equivalently, constants A, B, C, D, σ in (5.1) and (5.2).

According to preliminary simulations the following values of parameters appeared to be suitable ones:

$$A = B = 100; \quad C = 90, 100; \quad D = 90, 100, 110, 120; \quad \sigma = 1, 1.1; \quad \alpha = 50.$$

In all simulation tests there was kept

$$A = B = 100 \quad \text{and} \quad \alpha = 50.$$

Parameters C, D and σ were altered, to suit them for the exact layout of cities. For each considered configuration of parameters, 100 tests were done.

Results of simulations performed for the **city set 1** are presented in Tab. 1. In all tests, $\sigma = 1$ was used. The convergence rate of the network was very high (up to 100%) and did not at all depend on the initial state of the network. Best results were obtained for $C = 90$, $D = 110$. The average length in that case was only about 10% greater than the optimum. Lowering parameter D to 100 (with $C = 90$) resulted in 100% convergence with av. length 20% greater than the best one. Similar quality results were also obtained for the choices $C = 100$, $D = 120$ and $C = 100$, $D = 110$, respectively, but with lower convergence rate. Increasing parameter D to 130 caused the network to be unable to close tours. Trying to fulfil the minimization of (5.2) the network did not meet the requirement based on minimization of (5.1). On the other hand, setting $C = D = 100$ led to over 90% (100% for strategy P) convergence rate but the quality of tour lengths was evidently poorer.

Changing parameters C and D allows also adjusting the approximate average number of iterations. From Tab. 1, it can be seen that the higher the proportion $\frac{D}{C}$, the greater the average number of iterations required.

Table 1. Results of computer simulations for the city set 1.
Parameters: $A = B = 100, \sigma = 1$

PART	Best	Mean	Worst	% Succ.	Iter.	FULL	Best	Mean	Worst	% Succ.	Iter.
C=90, D=100						C=90, D=100					
a	2.785	3.19	3.69	100	80.7	a	2.767	3.21	3.79	100	121.1
b	2.696	3.22	3.65	100	64.4	b	2.765	3.21	3.88	100	107.0
c	2.696	3.17	3.68	100	64.4	c	2.696	3.21	3.70	100	97.1
d	2.696	3.20	3.63	100	51.1	d	2.696	3.21	3.69	100	106.3
C=90, D=110						C=90, D=110					
a	2.696	3.00	3.31	100	129.8	a	2.765	3.01	3.34	97	219.8
b	2.696	2.99	3.31	100	127.1	b	2.696	2.99	3.41	95	238.2
c	2.696	3.00	3.33	100	165.1	c	2.696	3.02	3.43	96	223.4
d	2.696	3.01	3.33	99	154.7	d	2.696	3.04	3.52	96	231.6
C=100, D=100						C=100, D=100					
a	2.877	3.52	4.38	100	33.4	a	2.937	3.49	4.08	97	51.8
b	2.765	3.51	4.21	100	34.5	b	2.787	3.53	4.51	95	65.5
c	2.785	3.54	4.42	100	31.4	c	2.785	3.45	4.14	93	68.7
d	2.696	3.55	4.20	100	33.9	d	2.767	3.47	4.15	97	61.4

Table 1 [cont.]

PART	Best	Mean	Worst	% Succ.	Iter.	FULL	Best	Mean	Worst	% Succ.	Iter
C=100, D=110						C=100, D=110					
a	2.696	3.24	3.88	92	107.3	a	2.696	3.22	3.89	82	157.3
b	2.696	3.26	3.89	93	113.5	b	2.765	3.27	4.05	79	159.2
c	2.696	3.26	3.88	93	137.4	c	2.696	3.22	3.79	78	155.4
d	2.696	3.23	3.88	94	109.9	d	2.696	3.25	3.75	81	116.3
C=100, D=120						C=100, D=120					
a	2.785	3.06	3.42	54	223.5	a	2.696	3.06	3.46	42	224.6
b	2.696	3.04	3.46	59	162.7	b	2.696	3.05	3.46	34	231.2
c	2.696	3.01	3.46	52	201.3	c	2.696	3.05	3.41	36	208.0
d	2.696	3.03	3.46	66	210.0	d	2.696	3.10	3.58	44	219.2

For the **city set 2** the same five sets of parameters were used (with $\sigma = 1$). Results are presented in Tab. 2. The convergence rate was also high — for $C = 90$, $D = 100$ about 95%, for $C = 100$, $D = 110$ and $C = D = 100$ over 80%. In the first two cases the mean length was about 20% greater than the best one. In the last case the average length was longer. For the two other sets of parameters the average length was much better, but the success rate was lower (especially for $C = 100$, $D = 120$). To improve the success rate (for $C = 100$) it was neccessary to lower parameter D to 90. In that case over 95% (100% for strategy P) convergence was achieved, but since the network "worked" mostly on the syntactical constraint (5.1), the mean length of the tours was about 3.65 which was 30% greater than the optimum.

The remark about the mutual dependence between the average number of required iterations and the proportion $\frac{D}{C}$ is also true for this set of cities.

Table 2. Results of computer simulations for the city set 2.
Parameters: $A = B = 100$, $\sigma = 1$

PART	Best	Mean	Worst	% Succ.	Iter.	FULL	Best	Mean	Worst	% Succ.	Iter
C=90, D=100						C=90, D=100					
a	2.862	3.24	3.91	99	123.9	a	2.862	3.22	3.89	94	188.3
b	2.862	3.21	3.91	96	139.1	b	2.862	3.26	3.82	97	174.6
c	2.862	3.23	3.69	98	109.6	c	2.862	3.26	3.93	97	189.3
d	2.862	3.21	3.87	95	127.4	d	2.862	3.25	3.95	98	220.8
C=90, D=110						C=90, D=110					
a	2.862	3.02	3.20	52	280.1	a	2.862	3.04	3.64	68	448.9
b	2.862	3.02	3.20	60	294.5	b	2.862	3.07	3.50	58	355.6
c	2.862	3.03	3.20	52	272.2	c	2.862	3.05	3.45	62	445.0
d	2.862	3.03	3.20	57	243.8	d	2.862	3.05	3.64	60	378.9

Table 2 [cont.]

PART	Best	Mean	Worst	% Succ.	Iter.	FULL	Best	Mean	Worst	% Succ.	Iter.
C=100, D=90						C=100, D=90					
a	2.880	3.66	4.55	100	41.3	a	2.919	3.65	4.28	97	54.5
b	2.919	3.71	4.69	100	33.9	b	2.989	3.65	4.62	95	68.1
c	2.996	3.64	4.39	100	38.4	c	2.880	3.69	4.62	98	70.9
d	2.862	3.66	4.51	100	47.5	d	2.862	3.46	4.38	99	66.4
C=100, D=100						C=100, D=100					
a	2.862	3.42	4.02	96	78.2	a	3.051	3.48	4.32	93	115.2
b	2.862	3.43	4.04	98	83.5	b	2.951	3.39	4.09	84	132.2
c	2.880	3.44	4.23	98	71.1	c	2.880	3.45	4.22	84	130.3
d	2.862	3.44	3.97	96	89.4	d	2.880	3.41	3.97	95	113.2
C=100, D=110						C=100, D=110					
a	2.880	3.26	3.76	82	93.5	a	2.880	3.32	3.89	68	145.7
b	2.862	3.31	3.89	85	113.1	b	2.880	3.31	4.04	65	143.0
c	2.880	3.32	3.85	81	139.1	c	2.880	3.28	3.81	64	204.0
d	2.938	3.34	3.85	89	116.5	d	2.862	3.25	3.87	66	175.3
C=100, D=120						C=100, D=120					
a	2.862	3.04	3.22	12	250.5	a	2.919	3.19	3.43	12	149.8
b	2.862	3.05	3.15	13	399.8	b	2.862	3.13	3.43	13	170.4
c	2.919	3.09	3.20	12	188.1	c	2.919	3.08	3.43	9	223.3
d	2.862	3.12	3.48	15	481.7	d	2.862	3.07	3.35	12	230.0

Table 3. Results of computer simulations for the city set 3.
Parameters: $A = B = 100$

PART	Best	Mean	Worst	% Succ.	Iter.	FULL	Best	Mean	Worst	% Succ.	Iter.
sigma =1						sigma =1					
C=90, D=90						C=90, D=90					
a	2.786	3.35	4.00	100	145.3	a	2.919	3.38	4.11	100	194.3
b	2.786	3.37	3.84	100	149.4	b	2.786	3.38	4.00	98	230.1
c	2.786	3.36	3.82	99	135.5	c	2.786	3.39	3.90	96	221.7
d	2.786	3.35	3.99	100	140.5	d	2.786	3.36	4.36	99	206.6
sigma =1						sigma =1					
C=100, D=100						C=100, D=100					
a	2.786	3.35	3.86	87	209.6	a	2.932	3.40	4.24	73	267.8
b	2.786	3.36	4.08	95	266.9	b	2.786	3.38	3.93	67	236.9
c	2.786	3.42	4.08	93	251.0	c	2.786	3.35	4.04	69	257.4
d	2.786	3.33	3.90	92	234.7	d	2.786	3.38	4.21	78	279.2
sigma =1.1						sigma =1.1					
C=90, D=100						C=90, D=100					
a	2.919	3.31	3.99	98	153.7	a	2.786	3.36	3.90	96	236.0
b	2.786	3.34	3.84	99	183.5	b	2.786	3.41	4.15	94	246.6
c	2.786	3.34	4.00	97	143.3	c	2.786	3.33	3.87	97	242.4
d	2.786	3.34	3.99	99	168.5	d	2.786	3.38	4.00	98	258.3

Solving the problem for the **third set** required slight modification of the parameters. In that case best results were obtained for $C = D = 90$ and $C = D = 100$, with $\sigma = 1$. Comparable results were also achieved for $C = 90$, $D = 100$, $\sigma = 1.1$ (Tab. 3). Generally, for this set of cities a little worse results were obtained, with the mean length about 20% – 25% greater than the shortest one.

Based on the three examples we can state that the network is quite sensitive to alteration of parameters. In all examples, when parameters had been changed beyond some limit values, either the lengths of tours have increased or the success rate has decreased or the average number of iterations has increased. The crucial for the method is the proper setting of proportions $\frac{C}{D}$ and $\frac{C}{A}$ (on condition that $A = B$). These proportions, rather than exact values, decide on the quality of results.

3. Final remarks and conclusions

The method of solving the TSP presented in this paper is based on some modifications of the "classical" Hopfield-Tank's approach. Primarily a new way of updating neuron output potential is applied. Based on initial simulations proper network constants were found, which led to good quality results, with mean lengths of tours about 10%–20% greater than the best lengths.

Based on the same method another non-plynomial problem — the NQP — was also solved ([9], [11]) with very good results.

Comparably worse results obtained for the TSP are due to stronger requirements associated with minimization of the tour length (5.2). In the NQP any valid solution is automatically the best one, since all solutions are equally good. On the other hand, because of additional constraints for all diagonals of matrix V in (5.1), the function of energy in the NQP is of a much more complicated form than that of the TSP.

Good results obtained for both problems indicate that after more research the proposed method may be applied to the wide subset of NP-Hard optimization problems.

The very good point of the introduced method is its independence on the initial state of the network. All four ways of setting the starting point of the network, resulted in similar quality of success rates and mean lengths of obtained tours.

There are, however, some open questions. In further research we plan to completely discretize the network, i.e. allow output potentials to take on values only from the set $\{0, 1\}$. We are not yet sure if the results would be of as high quality as they were for the NQP ([9], [10]).

Our interest is also associated with a theoretical analysis of an influence of network parameters on network bahaviour.

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